

Rhombuses, Rectangles, and Squares

VOCABULARY

Rhombus

a parallelogram with four congruent sides

Rectangle

a parallelogram with four right angles

Square

a parallelogram with four congruent sides and four right angles.

Example 1 Describing a Special Parallelogram

Decide whether the statement is *always*, *sometimes*, or *never* true.

- a. A square is a rectangle.
- b. A rectangle is a square.

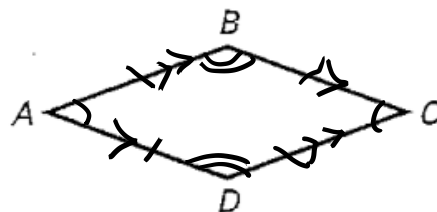
Solution

- a. The statement is always true. Because all squares have four right angles, squares are always rectangles.
- b. The statement is sometimes true. If a rectangle has four congruent sides, then it is also a square.

Example 2: Using Properties of Special Parallelograms

$ABCD$ is a rhombus. What else do you know about $ABCD$?

- $ABCD$ has four congruent Sides.
- Its opposite sides are parallel.
- Its opposite angles are congruent.
- Its diagonals bisect each other.
- Its consecutive angles are supplementary.



RHOMBUS COROLLARY

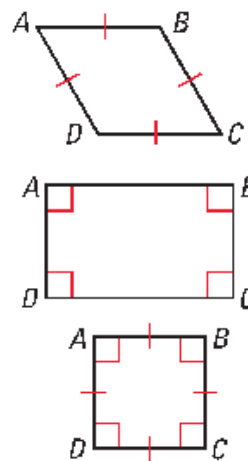
A quadrilateral is a rhombus if and only if it has four congruent sides.

RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

SQUARE COROLLARY

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

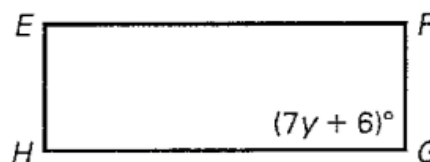


Example 3: Using Properties of a Rectangle

In the diagram, $EFGH$ is a rectangle.
What is the value of y ?

SOLUTION

All four angles of a rectangle are right.



$$7y + 6 = 90$$
$$y = 12$$

✓ Decide whether the statement is *always*, *sometimes*, or *never* true.

1. A rhombus is a square.

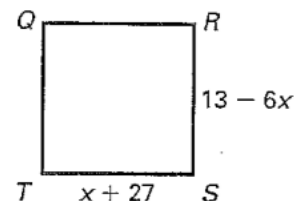
Sometimes

2. A parallelogram is a rectangle.

Sometimes

3. $QRST$ is a square.

What is the value of x ?

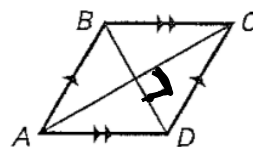


$$x + 27 = 13 - 6x$$
$$7x = -14$$
$$x = -2$$

THEOREM 6.11

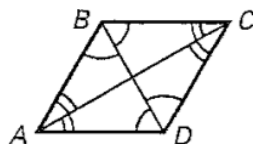
A parallelogram is a rhombus if and only if its diagonals are perpendicular

$ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

**THEOREM 6.12**

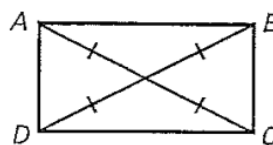
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle \underline{BAD}$ and $\angle \underline{BCD}$ and $\overline{BD}$ bisects $\angle \underline{ABC}$ and $\angle \underline{ADC}$.

**THEOREM 6.13**

A parallelogram is a rectangle if and only if its diagonals are congruent

$ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

**PRACTICE**

Decide whether the statement is *sometimes*, *always*, or *never* true.

1. A rhombus is equilateral. always
2. The diagonals of a rectangle are perpendicular. sometimes
3. The opposite angles of a rhombus are supplementary. sometimes
4. A square is a rectangle. always
5. The diagonals of a rectangle bisect each other. always
6. The consecutive angles of a square are supplementary. always

Quadrilateral $ABCD$ is a rhombus.

7. If $m\angle BAE = 32^\circ$, find $m\angle ECD$. 32°
8. If $m\angle EDC = 43^\circ$, find $m\angle CBA$. 86°
9. If $m\angle EAB = 57^\circ$, find $m\angle ADC$. 66°
10. If $m\angle BEC = (3x - 15)^\circ$, solve for x . 35°
11. If $m\angle ADE = (5x - 8)^\circ$ and $m\angle CBE = (3x + 24)^\circ$, solve for x . $x = 16$
12. If $m\angle BAD = (4x + 14)^\circ$ and $m\angle ABC = (2x + 10)^\circ$, solve for x . $x = 26$

